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SUMMARY OF EXPERIMENTAL ASTRONOMY LABORATORY WORK

June 1964

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INTRODUCTION

This report consists of reprints of talks given by members of the Experimental Astronomy Laboratory staff at the meeting on April 14, 1964, held at the Massachusetts Institute of Technology.

The papers describe in brief the work of our staff members for the last contract period, and outline work to be done in the coming year.

ATTENDEES OF APRIL 14, 1964 MEETING

Attendees:

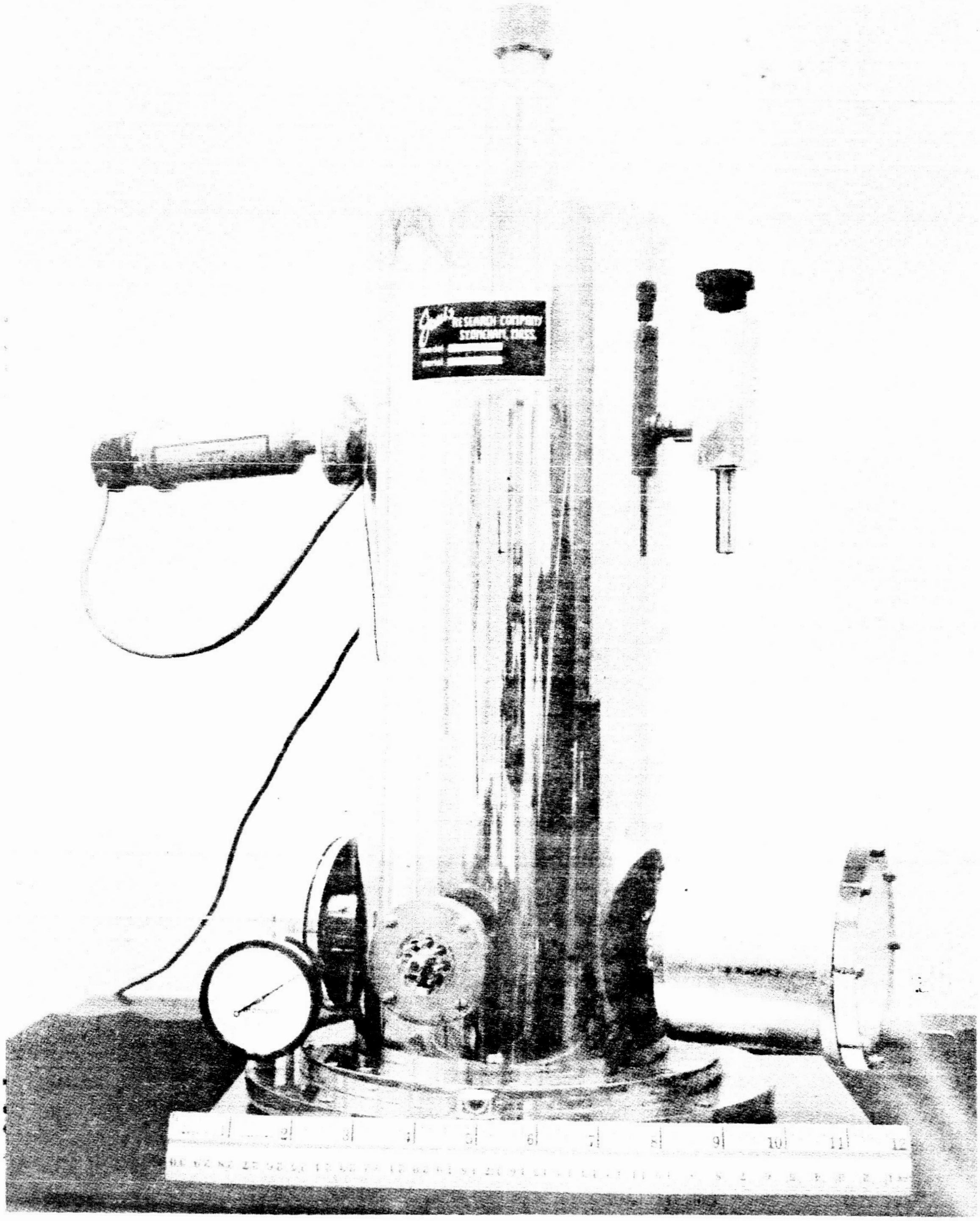
MIT

Experimental Astronomy Personnel

- B. Blood
- P. Chapman
- A. Conrod
- S. Ezekiel
- H. Fleischer
- E. Haddad
- W. Hollister
- W. Markey
- E. Mitchell
- J. Potter
- R. Stern
- L. Beckley
- H. Miller
- T. Egan
- J. Hendrickson
- J. Searcy

NASA Personnel

- Richard Hayes
- Roland Gillespie
- REG/Mr. L. Gilchrist
- Charles Pontius
- Jules Kanter
- Raymond Bohling
- RE-TG/Mr. A. J. Calio
- Eugene Darling
- G. Trafford
- RE-TG/Mr. S. Schwartz
- Thomas Burke



Frontispiece: Llama suspension dewar.

SECTION I

THE LLAMA PROGRAM

by

Philip K. Chapman

and

Shaoul Ezekiel

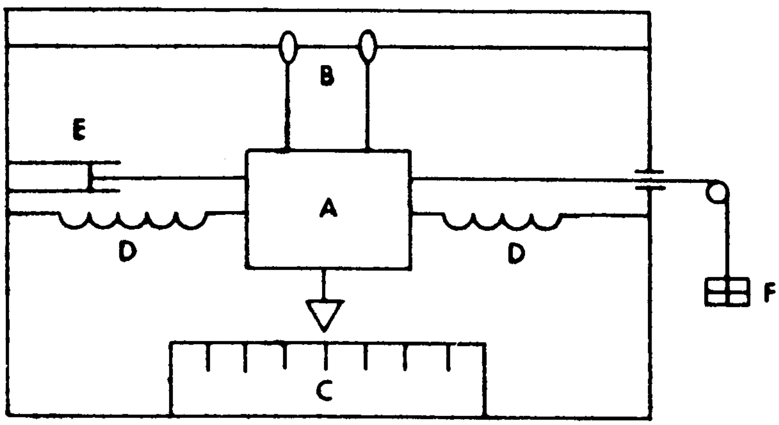
Introduction

LLAMA (for Low Level Acceleration Measurement Apparatus) is an investigation of techniques for refining the various components of a simple single-axis linear accelerometer, with the intention of constructing an exceptionally sensitive device which would be of use as a calibration standard in the test and evaluation of other instruments. The region of interest is below 10^{-3} cm/sec^2: the primary design goal is therefore a system which is absolute in the sense that it does not require the application of a known acceleration for calibration.

Subsystem Design

As shown in Fig. 1, an elementary linear accelerometer consists of five major components. Very stringent performance requirements are imposed on these components, especially the test mass suspension and the displacement readout, if the system is to operate satisfactorily at very low acceleration levels.

a) Test Mass Suspension: It is necessary to support the test mass in such a way that virtually zero stiction is exhibited when a small force is applied along the sensitive axis. Almost any mechanical support displays hysteretic effects at the levels under consideration, so various types of electromagnetic field suspension were investigated. Of these, the cryomagnetic (Meissner effect) suspension offered the lowest residual forces along the sensitive axis,



A. TEST MASS

B. TEST MASS SUSPENSION

C. DISPLACEMENT READOUT

D. RESTORING FORCE GENERATOR

E. DAMPING

F. CALIBRATION

Fig. 1 Components of a basic linear accelerometer.

and also appeared to be the simplest to mechanize.

It is well known that superconducting materials, being perfectly diamagnetic, are repelled by a magnetic field. This property is used in the cryogenic gyro, in which a superconducting sphere is supported by the field of currents flowing in adjacent coils. In LLAMA, the test mass consists of a small permanent magnet, which is stably suspended over a superconducting surface.

The phenomenon involved in this suspension is perhaps best understood by considering the case in which the superconducting surface is an infinite horizontal plane. Since the field of the small magnet cannot penetrate the superconductor, there can be no normal component of field at the surface. It is easy to see that the resulting field distribution is identical to that which would be created by the magnet plus an identical "image" below the surface. The float height is thus equal to half the separation at which the repulsion between two such magnets is just equal to the weight of one of them.

Since the image of course moves with the magnet, there can be no horizontal component of force from the superconductor. If, however, the superconducting plane is finite in extent, the field spills over at the edges, producing a force which tends to draw the magnet towards the nearest boundary. In other words, a finite planar suspension is slightly unstable with respect to horizontal motion.

In order to make a device which is sensitive only along a single axis, a superconducting tube rather than a plane is used in the LLAMA suspension. The magnet then encounters a restoring force on displacement in any direction away from the axis of the tube. Because of the edge effect mentioned above, the suspension is unstable with respect to axial displacement of the test mass from the center, but this instability can be reduced arbitrarily by lengthening the cylinder, or compensated as part of the design of the restoring force generator (see below).

In practice, it is necessary that the superconductor be singly-connected, since otherwise movement of the magnet would generate persistent supercurrents which would give the suspension itself a magnetic moment. The tube therefore has a narrow axial slit along the top.

A cross-section through the LLAMA suspension dewar is shown in Fig. 2. A thin-walled copper cylinder (O. D. 2. 2 cm) is inserted lengthwise in a hollow copper block, attached to the bottom of the inner vessel of the dewar. The superconductor is a sheet of niobium, wrapped around the outside of this cylinder, so that it is in contact with liquid helium when the dewar is filled. The ends of the cylinder are open to the dewar vacuum. In operation, the test mass floats just below the center line of the cylinder.

The transition temperature of all superconductors decreases monotonically with applied magnetic field. If the metal is cooled in a field which exhibits a local maximum of intensity, this region will remain normal after the rest of the material is superconducting, and any flux passing through it will be irrevocably trapped until the temperature is raised. The implication of this phenomenon in the present context is that the magnet must be external to the cylinder at the time the dewar is filled with liquid helium. The magnet is therefore stored in the room-temperature, evacuated antechamber shown at the left in Fig. 2. When the niobium is superconducting, the magnet is inserted into the suspension by means of a lucite spoon, which can be manipulated from outside the dewar via a vacuum feed-through.

The dewar is equipped with windows opposite the ends of the cylinder, which are used for observation of the test mass and for access by the displacement detection system.

b) Displacement Detection: Since the test mass moves very slowly at the acceleration levels under consideration, very small displacements along the sensitive axis must be detected if the instru-

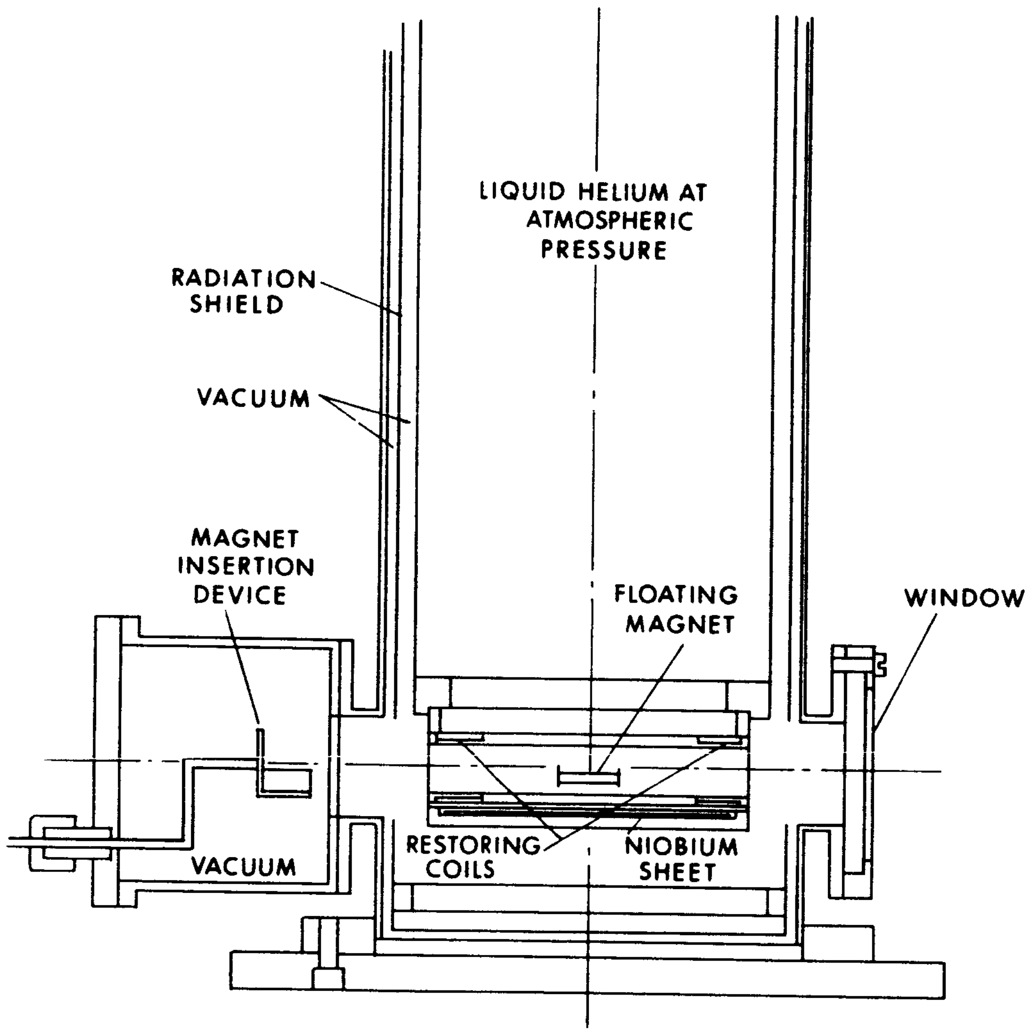


Fig. 2 Cross section of the LLAMA Dewar.

ment is to have a reasonable bandwidth. The precision required is such that interferometric methods offer the most practical solution.

A specialized Twyman-Green interferometer has been designed for LLAMA, using a CW gas laser as the light source. Small, optically flat and parallel mirrors are attached to the ends of the test mass, and these constitute the fundamental mirrors in a folded interferometric system. This technique gives twice the displacement sensitivity of a conventional interferometer and also allows the design of a system which is insensitive to the inevitable small oscillations of the test mass about axes perpendicular to the sensitive axis.

It is expected that this device will be capable of detecting displacements of the order of 0.1 micron.

c) Restoring Force Generation: As shown in Fig. 2, two small coils are located inside the niobium tube, near the ends. These coils are fed with direct current, in opposing series, in such a way as to restrain the magnet if it moves away from the center of the suspension. The magnitude of the current is adjusted until the suspension is almost neutrally stable.

In addition, the output of the displacement detector is used to command a differential current through the coils, in order to keep the test mass in the null position in the presence of applied acceleration. The magnitude of this differential constitutes the output of the accelerometer.

d) Damping: Eddy current generation in the copper tube surrounding the test mass provides substantial natural damping. The damping ratio for movement of the magnet is between 0.1 and 0.3, depending on the direction of motion. This is expected to be adequate, in view of the insensitivity of the displacement detector to motion except along the sensitive axis. Damping in that direction can of course be increased by filtering the signal in the restoring force feedback loop.

e) Calibration: For calibration of the instrument, a small, controllable and externally measurable force is required, which can be applied directly to the test mass. This is provided in the LLAMA system by radiation pressure from a 100-watt concentrated-arc mercury lamp. As the test mass weighs approximately one gram, a flux of 1.5 watts incident on one of the end mirrors imparts an acceleration of about 10^{-3} cm/sec^2.

The light from the calibration lamp is prevented from interfering with the operation of the displacement interferometer by suitable spectral filtering.

Present Status of LLAMA

Most of the effort to date has been directed at overcoming practical problems in the construction of the test mass suspension, which is now operational on a routine basis. Some preliminary experiments, aimed at verifying the design of the interferometer, have been undertaken with encouraging results, and a suitable optical system has been constructed for concentrating the light from the calibration arc lamp on to the small mirrors at the end of the test mass.

Pending completion of the interferometer, the restoring force feedback loop has been closed, using a relatively crude optical displacement sensor.

In addition to these experimental studies, a theoretical analysis has been made of the field distribution inside the suspension, the object being to estimate the ultimate sensitivity which could be reached with the system. Apart from the end effects, the limiting factor appears to be surface roughness and distortion of the superconductor. This effect is not as important in LLAMA as in some other field-supported devices, because the inertial element floats at a height of about 1 cm, as compared with the clearances (of the order of 0.01 cm) which are used, for instance, in the electrostatic gyro. It is expected that, with careful construction, the instrument might be useful down to the

10^{-6} - 10^{-7} cm/sec^2 range.

Conclusion: The Direction of Future Work

Once the present instrument has been assembled and thoroughly tested, there are several improvements which will be investigated. In particular, the use of a vacuum-deposited niobium film over the outside of a finely lapped copper cylinder should substantially reduce the surface-roughness limit on performance. In addition, it should be possible to compensate for end effects in the suspension by forming the outside of the cylinder to a slightly bulbous shape before depositing the niobium.

The first application of the instrument is expected to be as the sensor in a system for dynamic levelling of a table: i.e., an active filter for attenuating microseisms.

If LLAMA proves to be sufficiently sensitive (e.g., a threshold below 10^{-5} cm/sec^2), the gravitational field of a nearby, swinging mass might be used as an alternative calibration input.

Finally, for space application of this type of system, it would be possible to use a non-superconducting diamagnetic suspension, such as a bismuth tube, as long as the transverse field does not exceed about 5 x 10^{-4}g.

SECTION II
STATISTICAL TECHNIQUES FOR INERTIAL GUIDANCE COMPONENTS

by
Henry Fleischer

Ideally inertial guidance components should produce an output which is directly proportional to the input. In the case of an accelerometer, the output should be proportional to the input acceleration. Or, in the case of a gyroscope the output should be proportional to an applied torque. This, however, is not true; the actual output of an accelerometer may be represented by a power series plus some noise and the output of a gyroscope by a fourier series plus some noise. It is the function of testing to define the coefficients of the power series and the fourier series in a simple and reliable fashion.

At present, the basic tool for reducing gyro and accelerometer data is a fourier analysis. For a gyro the input axis is aligned parallel to the polar axis and the angular precession of the gyro is compared to Earth's rate. The output could then be synthesized by a harmonic series.

Harmonic Series:

$$\omega \text{ out} = C_0 + C_1 \sin\theta + C_2 \cos\theta + C_3 \sin 2\theta + C_4 \cos 2\theta + \text{Noise},$$

where θ is the turntable angle.

For an accelerometer, the input axis would be placed in vertical plane and varied with respect to gravity sinusoidally. The output from the accelerometer would then be represented by a harmonic series. The coefficients determined in the harmonic series could be directly related to those of a power series and thus the instrument parameters are defined.

Power Series:

$$A_{out} = K_o + K_1 A_{in} + K_2 A_{in}^2 + K_3 A_{in}^3 + \ldots$$

Harmonic Series:

$$g_{out} = C_o + C_1 \sin\phi + C_2 \cos\phi + C_3 \sin 2\phi + C_4 \cos 2\phi + \text{Noise},$$

where ϕ is the angle between the Input Axis and the vertical.

A fourier analysis readily evaluates the coefficients of a harmonic series. However, it does not define the stability of the coefficients nor does it define the properties of the noise, i.e., be it of random or systematic nature.

It is hoped that via the use of correlation techniques, the nature of the noise could be determined as well as the stability of the coefficients.

As a first approach to this it was decided to utilize the 1620 Computer and program cross correlation and auto-correlation functions. The auto-correlation function may be represented by:

$$Rxx\,(J) = \frac{1}{2\pi} \int_o^{2\pi} (X_I X_{I+J})\, dI$$

for continuous data and differentially spaced data points. This may be replaced by

$$Rxx\,(J) = \frac{1}{N-J} \sum_{I=1}^{N-J} X_I X_{I+J} \qquad \text{(See Fig. 1)}$$

for finite data:

Cross correlation programs for sin (I+ J) and sin (2I+ J) were established

$$Rxy\,(J) = \frac{1}{2\pi} \int_o^{2\pi} (X_I Y_{I+J})\, dI$$

$$Rxy\,(J) = \frac{1}{N-J} \sum_{I=1}^{N-J} X_I Y_{I+J} \qquad \text{(See Figs. 2 and 3)}$$

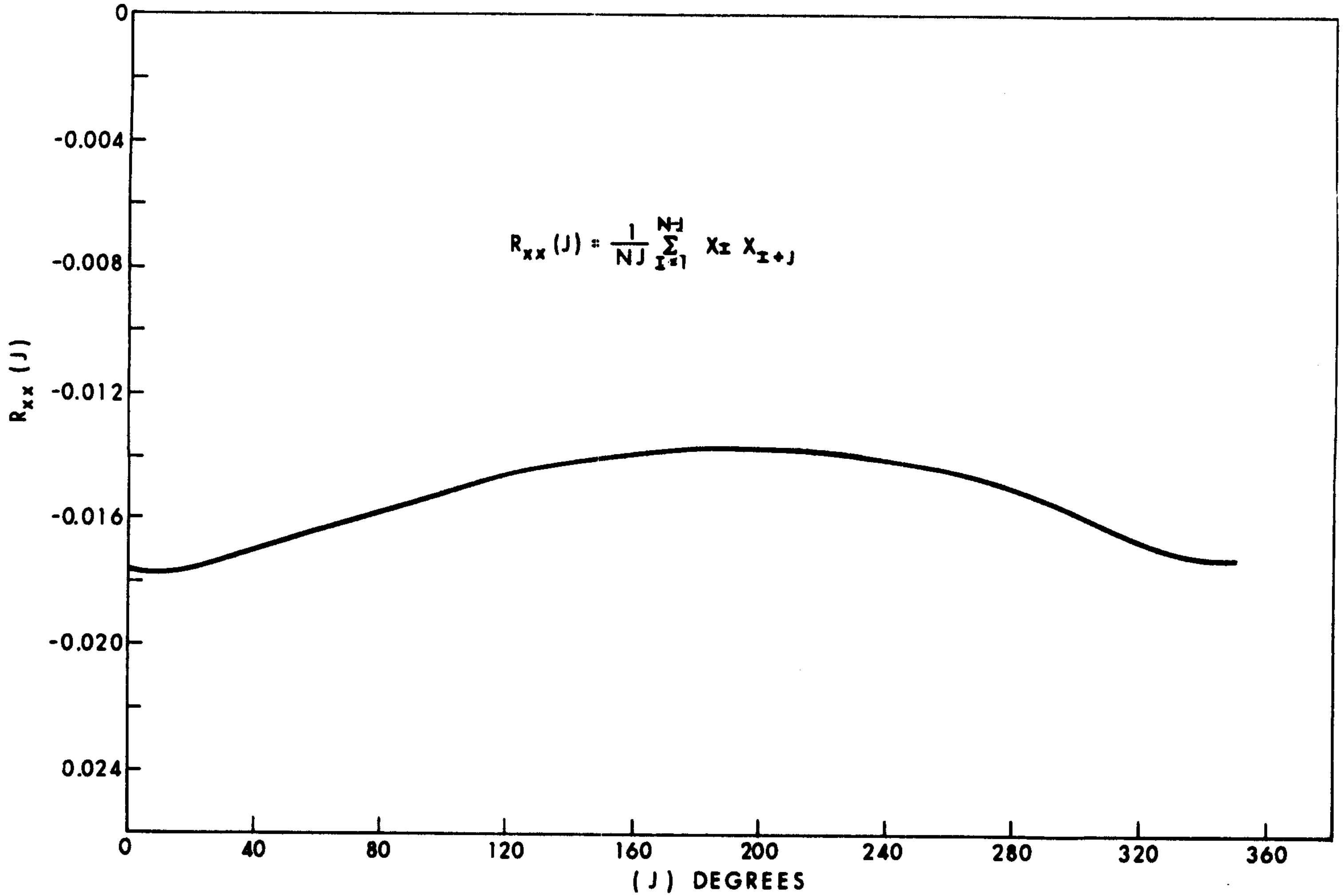


Fig. 1 Auto-correlation curve.

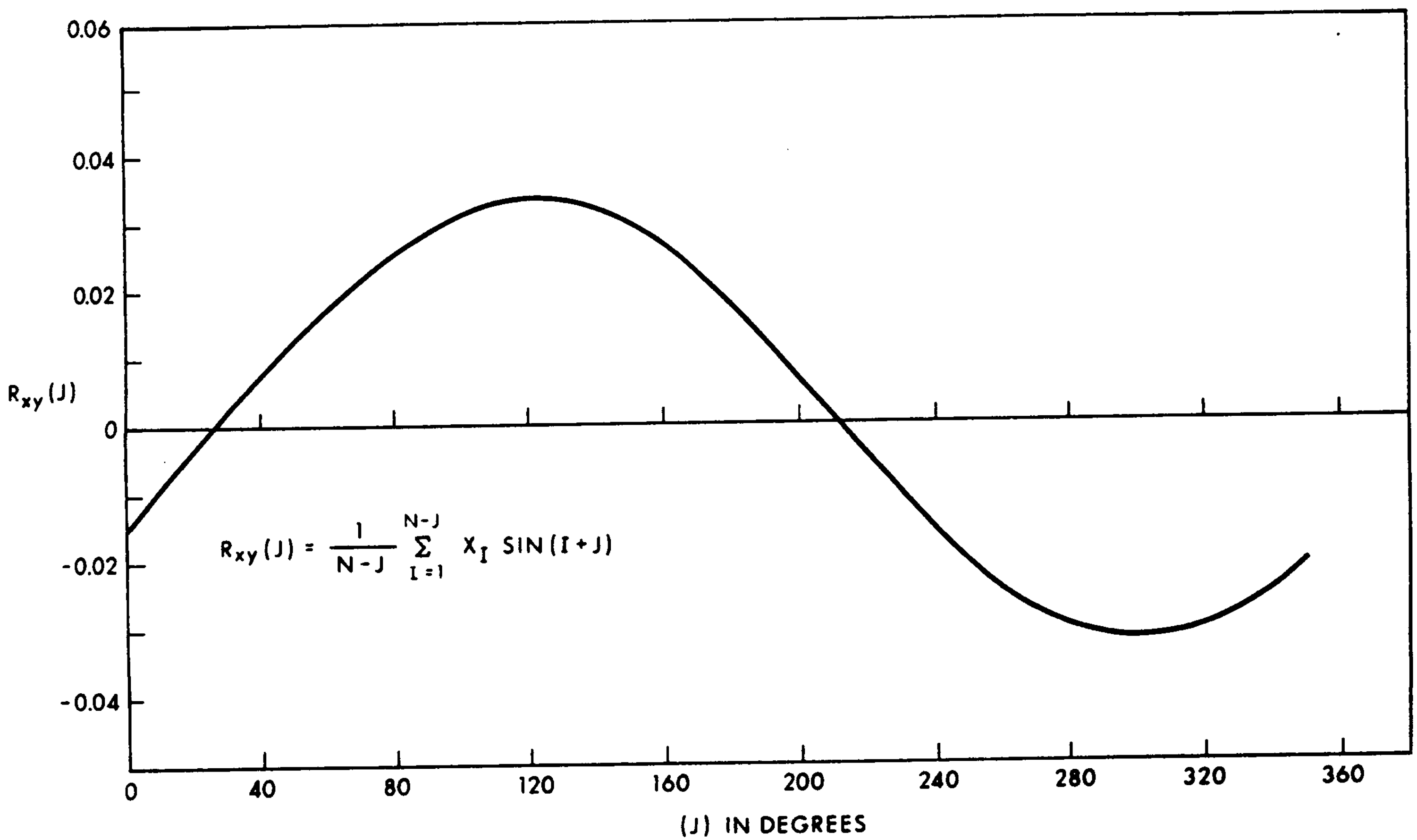


Fig. 2 Cross correlation curve of sin (I + J).

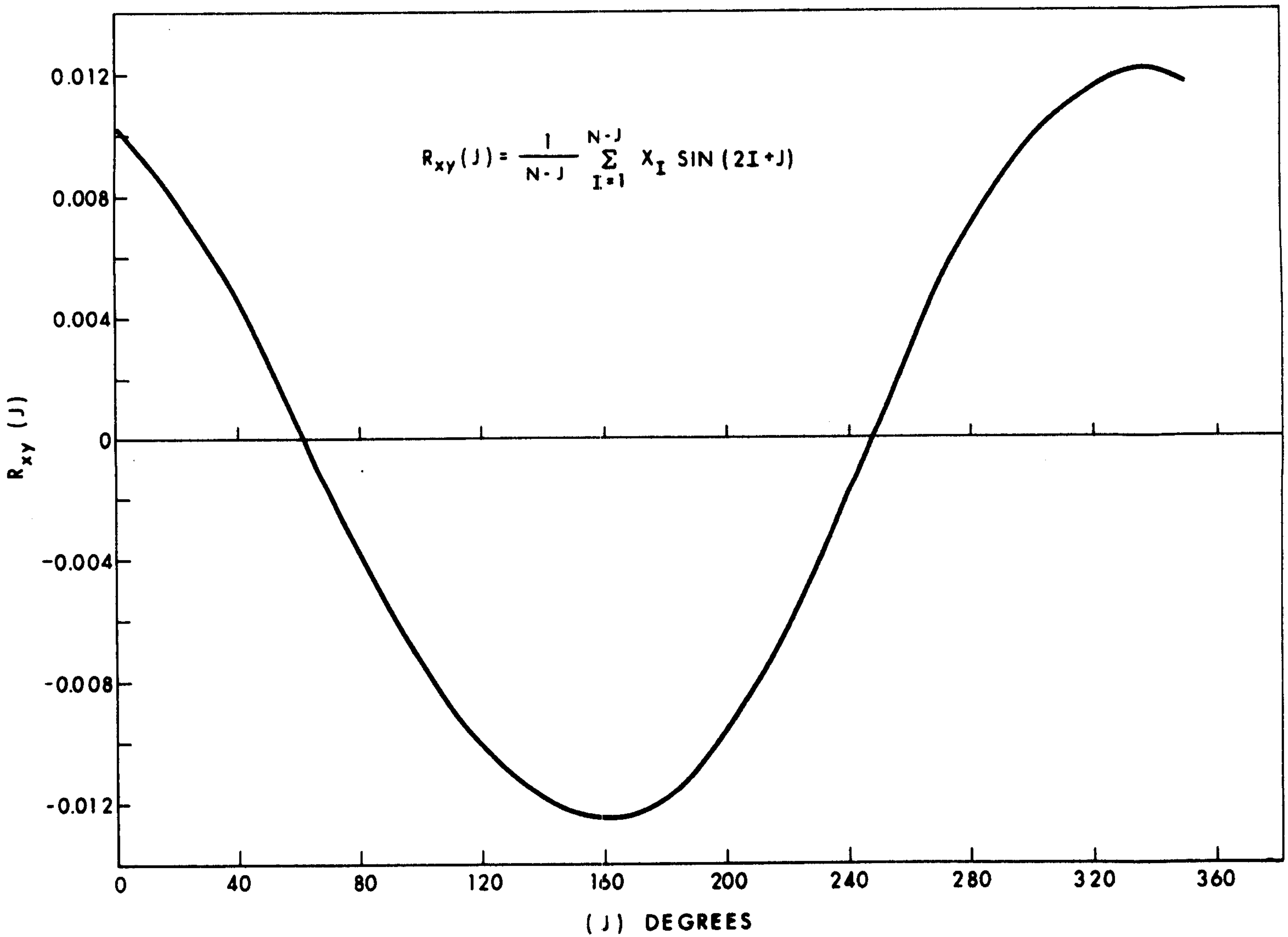


Fig. 3 Cross correlation curve of sin (2I + J).

It was desired to obtain a complete curve of both auto-correlation and cross correlation fuctions for 360° spaced at 10° intervals. In order to have 98% reliability of the coefficients, it was determined that 10 runs of data spaced every 10° were required. As a check the coefficients determined via the utilization of auto-correlation and cross correlation techniques could be compared to fourier analysis coefficients.

For an auto-correlation program:

$$R_{xx}(J) = (C_1^{\;2} + C_2^{\;2}) \cos J + (C_3^{\;2} + C_4^{\;2}) \cos 2J + C_o^{\;2}$$

For cross correlation programs:

a) $Y_{I+J} = \sin (I + J)$

$$Rxy\ (J) = \frac{1}{2} (C_1 \cos J + C_2 \sin J)$$

b) $Y_{I+J} = \sin (2I + J)$

$$Rxy(J) = \frac{1}{4} (C_3 \cos J + C_4 \sin J).$$

The above equations neglect the effect of noise. Graphs of the proceeding have been obtained and the coefficients compared. This is shown in the following box.

	Correlation Coefficient	Fourier Coefficient
C_o	-.0855 to -.1395	-.124
C_1	-.0276 to .0356	-.049
C_2	.0512 to .0508	.067
C_3	.0424 to .0476	.047
C_4	-.0192 to -.0220	-.006

The table above was obtained utilizing one revolution of gyro data which was assumed symmetrical for ten revolutions. Obviously, the previous assumption is not valid. This points to the need of obtaining further data. It is felt that ultimately this technique could lead to further understanding of the nature of noise in inertial guidance components.

SECTION III

NOISE IN OPTICAL LINE OF SIGHT DETERMINATION

by

Alfred C. Conrod

The Experimental Astronomy Laboratory has been studying the feasibility of daylight star tracking with photomultiplier tubes on a separate contract.

The conditions of measurement are shown in Fig. 1 which indicates the daytime sky brightness background, representative stellar spectra, and a detector response curve. The relative magnitudes of the signals from the stars and from the sky background made a direct measurement of star signals impossible, so we have attacked the problem of daylight star tracking with analytical and experimental studies to find ways of improving signal-to-noise ratios by applying spatial and spectral filtering to the input and electrical data processing techniques to the photoelectric cell output.

So far, our experimental emphasis has been on spectral filtering, partly because we have less prior information on this problem, and partly because the relative spectral curve shapes have suggested it.

We suggest an extension of this approach, that is, the investigation of the techniques of measurements, to the general study of optical measurements pertaining to space navigation.

Figure 2 shows a schematic model of an optical measurement. We will attempt to determine the effects of input and instru-

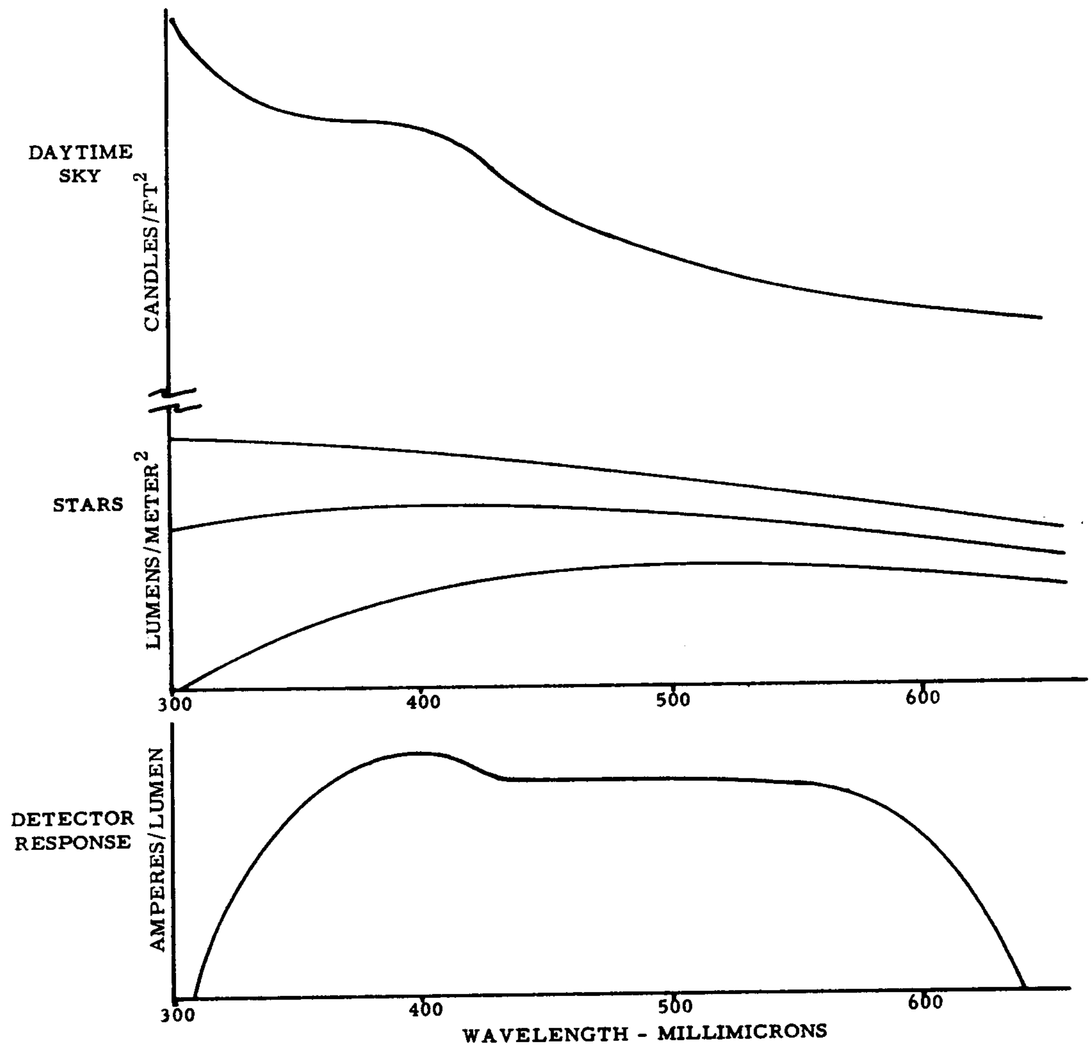

DAYTIME
SKY
CANDLES/FT²
STARS
LUMENS/METER²
DETECTOR
RESPONSE
AMPERES/LUMEN
300
400
500
600
WAVELENGTH - MILLIMICRONS

Fig. 1

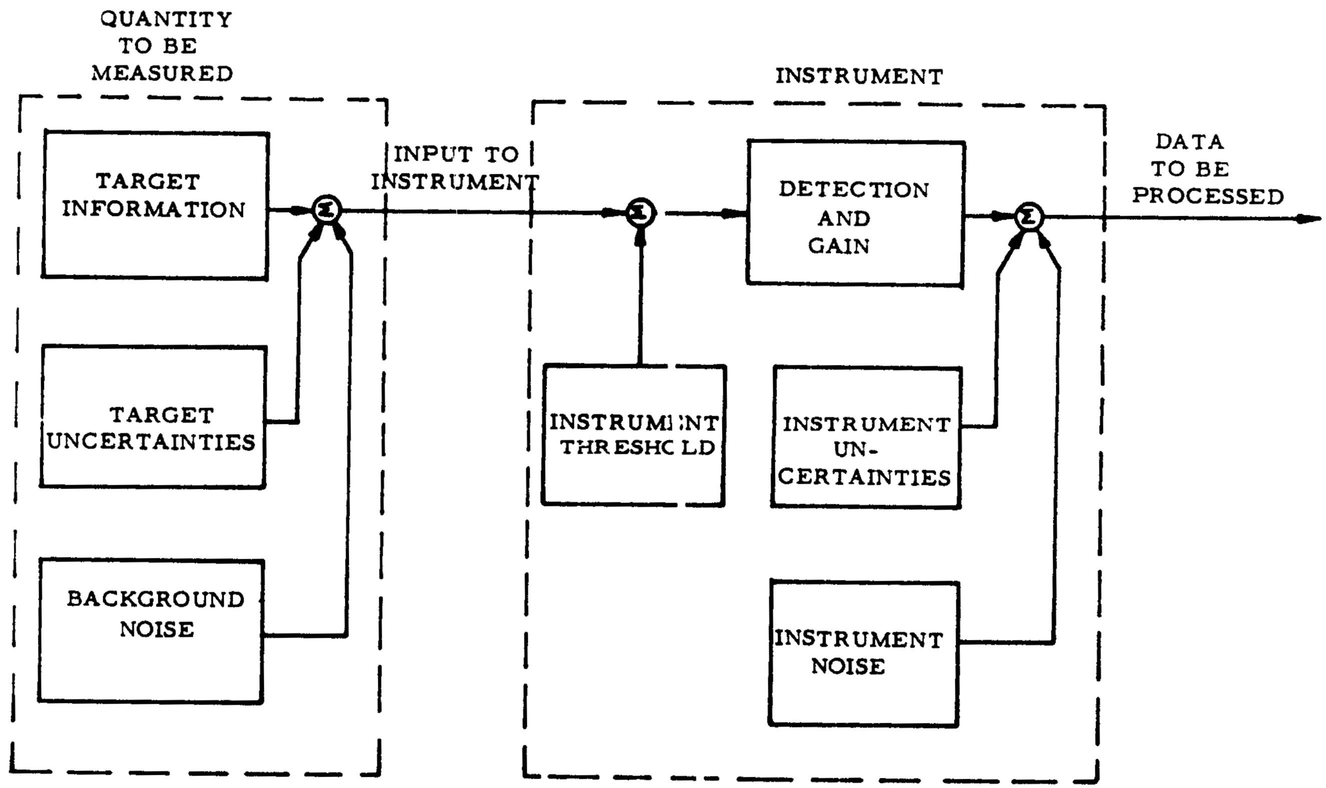

QUANTITY
TO BE
MEASURED
TARGET
INFORMATION
TARGET
UNCERTAINTIES
BACKGROUND
NOISE
INPUT TO
INSTRUMENT
INSTRUMENT
INSTRUMENT
THRESHOLD
DETECTION
AND
GAIN
INSTRUMENT
UN-
CERTAINTIES
INSTRUMENT
NOISE
DATA
TO BE
PROCESSED

Fig. 2

ment uncertainties and noise, and instrument thresholds on instrumant performance, especially with regard to defining those quantities so that the performance of instruments of different classes can be compared to common criteria for a given set of input conditions. In order to isolate and identify these effects, we will design experiments in which the test conditions, test instrumentation, and environment remain fixed and the instruments under test are exchanged, or their operating parameters modified

The test results should show the uncertainties in determination of the line of sight to a star, planet limb or planet landmark caused by noise in the target inputs and in the sensors, giving the instrument and systems designers valuable data to aid them in their work.

Evaluation of the test data can be accomplished in part with the correlation machine mentioned by Mr. Fleischer, and the results of the electro-optical test program should provide a valuable complement to the work on inertial components that he has described.

SECTION IV

PROGRESS REPORT - GUIDANCE ANALYSIS
JANUARY 1 TO MARCH 31, 1964

by

Robert G. Stern

The primary objective of the Guidance Analysis group during the first few months of this year has been a numerical evaluation of several simplified mathematical models for achieving interplanetary midcourse guidance. The standard model, to which each of the simplified models is being compared, is a many-body model for which the equations of motion are integrated numerically on the IBM 7094 computer. The computer program is one that was prepared for NASA by JPL. The program is very elaborate and requires special handling on the MIT Computation Center computer. After a number of frustrating experiences with this program, we now have it running well, and we can generate the trajectories we need when computer time is available.

The three simplified models we are examing are the following:

1. Two-body nonlinear
2. Many-body linear
3. Two-body linear

The two-body nonlinear problem is being handled by Robert Gielow, who will complete an S. M. degree thesis based on it this semester. Two computer programs have been written; the first determines the position of a "phantom" target based on the characteristics of the osculating conic at the time of the midcourse correction, and the second determines the velocity correction required to arrive at the target when the position variation at the time of the correction is known. Both programs are now working, and velocity corrections are being generated.

The many-body linear problem has been assigned to Toru Tanabe. As a consequence of the linearity the problem can be set up in matrix form. The matrix coefficients are obtained by numerical integration backward in time from the destination point. The computer program being developed takes its inputs from the output of the JPL Trajectory Program at each integration interval. The basic program has been written, and work is now proceeding on the adaptation of the JPL output data. Program debugging cannot proceed until the adaptation has been completed. The program has considerable versatility, in that it can generate two-body solutions as well as many-body solutions and hence can show the effect of the added bodies on the computed corrections. The two-body solution obtained by numerical integration can be checked against the two-body analytic solution generated by the third simplified model.

The third simplified model is the two-body linear model, which is presently being studied. The program for this model is relatively straightforward. It is based on the analytic solution of the linearized variant equations. Velocity corrections are now being generated. Various schemes are being tried to minimize the errors caused by the gravity field of the destination planet.

To date no data have been obtained to determine the accuracy of the various midcourse corrections. These data are to be obtained by numerical integration using the JPL program. The big bottleneck at the present time is the unavailability of adequate computer facilities. The JPL Trajectory Program requires special handling on the 7094, and the Computation Center apparently is unable to supply us with the amount of special running time that we need. To achieve a partial solution of this dilemma, our people have agreed to come in at 8 a. m. on Saturdays and/or Sundays when special handling will be made available to us (we hope) for a half hour. We feel that this can serve only as a temporary expedient; a more formal computer policy would be highly desirable.

In addition to the investigation of the guidance models, a report has been written on a simple method of selecting the stars to be used in optical sightings for the purpose of determining vehicle position on an interplanetary flight. The method is very much simpler than previous methods, and in the one sample calculation that was made its accuracy was roughly equivalent to that of the more elaborate methods.

SECTION V

SIMPLIFIED CONCEPTS OF INTERPLANETARY NAVIGATION

by

Walter M. Hollister
and
Robert Stern

1. Background

Since January 1964 the authors have made the following specific accomplishments in the area of interplanetary navigation:

1) the introduction of a new graduate course titled, "Special Problems in Interplanetary Flight."
2) the supervision of six graduate theses.
3) the formalization of some simple concepts of interplanetary navigation into a set of printed notes.

The new graduate course is an experimental one designed primarily to stimulate research in this field. There are 13 regular students in addition to 9 listeners some of whom are Experimental Astronomy Laboratory personnel. The course is taught by Prof. Hollister, Dr. Stern, and LCDR Mitchell and they present material from their respective doctoral theses. Six of the students are also working on graduate theses. The names of the six are Carlson, Gielow, Holbrow, Ruth, Munnell and McFarland. Carlson is writing an Engineer's thesis on the application of low-thrust VTA guidance theory to the generation of optimal variable-time trajectories. He hopes to finish in June 1964. Gielow is writing a Master's thesis which is an evaluation of the two-body, non-linear model for interplanetary navigation. He should finish by June having completed a good portion of the work already. Holbrow is writing a Master's thesis on the selection of the best celestial bodies for optical sightings. He expects to finish by June and is off to a good start. Ruth is attempting to formalize interplanetary navigation into the classical fire-control problem to see if it offers any simplification. He is just getting started and does not plan to finish until the Summer term. He is working now for a Master's degree but will continue on as a doctoral candidate next year.

Munnell is starting a Master's thesis which will extend Stern's work to include motion around a hyperbolic reference trajectory. He plans to continue the work through the Summer. McFarland is registered for only three units of thesis. He plans to select a topic for a Master's thesis within 3 weeks and complete it during the Fall term.

A copy of the work referred to in 3) follows below. It shows the first results of applying simple models to the problem of interplanetary navigation. In addition to giving a clearer physical picture of what is going on there is the possibility that some models may give sufficient accuracy for actual navigation. This will be tested along with the other navigation methods once the JPL trajectory generating program is running smoothly on the MIT 7094 computer.

2. Introduction

The objective of this work is to present the simple, basic principles involved in the navigation of a manned space ship during an interplanetary trip to Venus or Mars. The reason for providing this material is because the problem of interplanetary navigation is usually presented with so much complex mathematics that the basic physical principles are obscure or impossible to visualize. The problem is investigated here with simple, approximate models which demonstrate the predominant physical principles even when the models are insufficiently accurate for actual navigation. The value of this approach is that the problem is stripped of its complexity so as to show the major contributing factors in clear perspective. When a simple model is sufficiently accurate for navigation, system complexity is avoided. Even when it is not, the model allows numerical sizing of the parameters and shows the trade-off between accuracy and complexity. For the student, the engineer, or the human astronaut charged with the responsibility for interplanetary navigation these simple models should give a basic understanding of the problem.

3. The Interplanetary Trip

The interplanetary trip is expected to start from a parking orbit near Earth. A hyperbolic escape maneuver is performed which puts the spaceship on a free-fall trajectory toward the target planet. The impulsive velocity change for the maneuver occurs near the periapse of the escape hyperbola. If the velocity impulse could be applied without error, the spaceship would free-fall to the destination without further concern. Because the velocity impulse cannot be applied perfectly, the spaceship needs additional correctional velocity changes. The determination of the magnitude, direction and timing of these correctional velocity changes constitutes the interplanetary navigation problem. Once started on the interplanetary trajectory, position is determined by measuring the line of sight to selected points in the solar system relative to the star background. Velocity is determined from position information by carrying an accurate clock on board. An explicit transformation of the measured angles into the navigational velocity correction involves such gross computational complexity that the problem is usually linearized about a reference trajectory. In this manner the vector of deviations in the measured angles relative to the reference angles can be transformed into the navigational velocity correction by means of a single matrix multiplication. The value of the reference angles, the time at which they are to be measured, and the matrix which transforms the measured deviations into the velocity correction are all stored on board the spaceship. Because the celestial angles cannot be measured without error, the computed velocity correction is never perfect and subsequent sightings and velocity corrections are needed to null the error arising from previous sightings. In general, a miss at the target is corrected with the least velocity if done at a long distance from the target. On the other hand, the sighting accuracy is best at a short distance from the target. The only correction which directly affects the miss distance is the last correction. This means that to conserve fuel the last correction will be at as large a distance from the target as the sighting accuracy can allow while still providing an acceptably small miss distance. All other corrections are made only to minimize the expected value of the total required velocity correction. It is noted again that only the last correction has any effect on the miss distance and the primary purpose of all the other corrections is to minimize the expected value of the total correction. After the last correction the spaceship free-falls on a hyperbolic approach path to the destination planet. The interplanetary trip terminates near the periapse of the approach hyperbola by entering the planet's atmosphere for braking, by retrothrusting to enter a parking orbit, or by starting a new interplanetary trip with a direct entry onto an outgoing escape hyperbola.

4. The Observer's Point of View

The character of the motion which takes place during an interplanetary trip depends on the point of view of the observer. If the observer uses a frame centered at the Sun and non-rotating with respect to inertial space, the departure planet, the spaceship, and the destination planet all appear to be in elliptic orbit about the Sun. The acceleration of any freely-falling body due to the Sun in this frame is about 6×10^{-3} g at Earth's orbital radius. The acceleration due to the Sun at the orbital radius of Venus is about twice as large while at the orbital radius of Mars it falls to about half of the value at Earth's orbital radius. If the observer uses a non-rotating frame centered at the Earth, the character of the motion is quite different. The observable acceleration due to the Sun now consists only of the vector difference between the acceleration of the spaceship relative to the Sun and the acceleration of the Earth relative to the Sun. In general, the acceleration, $\ddot{\overline{R}}_{EP}$, of a free-falling spaceship at a point, P, relative to a non-rotating Earth-centered frame is given from Newton's Law by

$$\ddot{\overline{R}}_{EP} = \overline{G}_{EP} + \sum_{B} \left(\overline{G}_{BP} - \overline{G}_{BE} \right) \qquad (1)$$

where $\overline{G}_{EP}$ is the gravitational force per unit mass acting on the spaceship at P due to the Earth, $\overline{G}_{BP}$ is the gravitational force per unit mass acting on the spaceship at P due to any other body B and $\overline{G}_{BE}$ is the gravitational force per unit mass acting on the Earth due to that other body. At the surface of the Earth the magnitude of $\overline{G}_{EP}$ is one g and the magnitude of $\overline{G}_{SP} - \overline{G}_{SE}$, the contribution due to the Sun, S, is less than 10^{-7} g. At further distances from Earth $\left|\overline{G}_{EP}\right|$ falls off inversely as the distance squared while $\left|\overline{G}_{SP} - \overline{G}_{SE}\right|$ increases approximately linearly with the distance. At 400 Earth radii out $\left|\overline{G}_{EP}\right|$ has fallen below 10^{-5} g and $\left|\overline{G}_{SP} - \overline{G}_{SE}\right|$ has grown to about 10^{-5} g, both of which are very small accelerations. At this distance from Earth the acceleration of a freely-falling spaceship is one hundred times smaller in an Earth centered frame than in a Sun-centered frame. It should not be surprising therefore, that an interplanetary trip looks in a non-rotating Earth-centered frame like straight-line motion for a few months after Earth escape. The line of sight to the spacecraft for instance does not change much more than 15 degrees in the first 90 days of a typical trip. If the trip is observed in a non-rotating frame centered at the destination planet, the spaceship exhibits straight-line motion during the final portion of the trip prior to capture. Breakwell et al (1) have noted this phenomena and suggested that all the navigation be performed near the launch planet and the destination planet so that the straight-line motion can form the basic model for the navigation. They show with sample calculations that this assumption causes only an extremely small increase in the required velocity. What is really suggested is that the best point of view to take is that of the observer in the spacecraft itself. During the time when the navigation is going on, the planet of interest moves relative to a non-rotating frame centered in the spaceship as if there were practically no forces acting. Since the motion of interest to the observer in the spaceship appears to approximate unaccelerated motion in field-free space, it seems justifiable to look at inertial motion as the simplest model to demonstrate basic principles. It should be cautioned that the principles derived from the model can only be expected to be valid when navigating close to the departure planet or close to the arrival planet.

5. Practical Considerations

In order to develop a feeling for the size of the physical quantities involved, numerical values are given for the various parameters. Numbers quoted should be considered as representing order of magnitude only. In the case of instrumentation accuracy the numbers represent an approximate conservative prediction of what can be expected. The two most convenient units of distance for visualization are the astronomical unit, au, and the Earth's radius. For comparison the radius of Venus is approximately the same as Earth's while the radius of Mars is about one half Earth's radius. One au is about 2.4×10^4 Earth radii. The accuracy of the sextant is expected to be about 10^{-4} radians which is about 20 arc-seconds. This is approximately the angle subtended by the Earth's diameter at a distance of one au. A typical manned interplanetary trip takes about 150 days and finds the departure and arrival planets about 2/3 au or 15,000 Earth radii apart at arrival. This means that after escape and prior to capture typical relative velocities are the order of 100 Earth radii per day. The original injection velocity is expected to be applied with an accuracy within one part in one thousand. This means an uncorrected initial error causes a miss at the target of the order of 15 Earth radii. Subsequent navigational corrections are expected to be applied with an accuracy within one part in one hundred. After original injection the velocity error is about one part in one thousand. If the first correction were determined perfectly from perfect optical sightings, the accuracy of the application of the velocity correction would reduce the total velocity error to one part in 10^5. The sighting accuracy however in only good to one part in 10^4 so that after the first correction the total velocity error depends primarily on the sighting inaccuracy. The accuracy required at the destination can be assumed to be of the order of the accuracy required for atmospheric entry. Typical entry corridor widths are the order of 20 miles at Earth, so let the acceptable miss distance be of the order of 5×10^{-3} Earth radii. This means that the last correction will be on the order of 50 Earth radii out from the destination planet.

So far the only errors considered have been due to imperfect velocity application and optical sighting. There are several other errors which should be considered. The error in the clock for instance is expected to have a negligible effect by comparison with the 10^{-4} error in the optical sightings. A 10^{-4} drift rate would give an error of about 20 minutes after 150 days. Clocks have been built with drift rates several orders of magnitude better than 10^{-4}. Furthermore, the clock can be corrected from celestial sightings or time checks from Earth. The time which light takes to travel the astronautical distances must be considered although it does not constitute an error because it is a known correction which is included in the computation of the reference angles. It takes about 8 minutes for light to travel one au. The radiation pressure from the Sun is an important consideration. Because it applies a force in the direction opposite to the Sun's gravitational force, it effectively decreases the gravitational constant of the Sun by an amount depending upon the mass and reflecting characteristics of the spaceship. JPL[2] gives this factor as 2×10^{-5} for a 300 kilogram spaceship with a perfectly reflecting area of 3.6 square meters. This is equivalent to a force differential of between 10^{-7} and 10^{-8} g at the Earth's orbital radius and causes an error of a few planet radii at the destination if left uncorrected. The reference trajectory would account for the expected effect but variations from the anticipated radiation pressure add an uncertainty which requires navigational correction. The error in our knowledge of the magnitude of the astronomical unit in terrestrial units causes a direct error in the computed velocity increments. This error is expected to be much less than one part in 10^4 by the time manned missions are attempted. The aberration of starlight due to relative motion of the observer is of the order of 20 arc-seconds, and must be considered. Again, it does not represent an error because it is taken care of in the reference angles. Stellar parallax is 30 times smaller than aberration and may be neglected. In summary, the computation of the reference angles must take into account the aberration of light, the time of light propagation, and the effect of solar radiation pressure. The major error in the reference is expected to be the uncertainty in the exact solar radiation pressure. After initial injection into interplanetary orbit, the major residual error will be the one part in one thousand error in applying the initial injection velocity. The first velocity correction after injection will remove the majority of the initial velocity error. All corrections after initial injection will leave residual error due primarily to the inaccuracy in the celestial sightings of one part in 10^4. The last correction, made about 50 Earth radii from the destination, will reduce the expected miss to about 20 miles.

6. Velocity Space

The determination of a unique trajectory between two planets is completely defined by the specification of a launch date and an arrival date. The launch date fixes the launch position by the position of the launch planet at the launch date and the arrival date fixes the arrival position by the position of the arrival planet at the arrival date. There is only one practical free-fall trajectory which passes through the two points and has the given time of flight. (The trajectory which goes around the Sun in the retrograde direction is not considered practical.) Once a unique trajectory corresponding to the given launch and arrival dates has been found, it is straight forward to compute the incremental inertial velocity vector required to put the spaceship on the computed trajectory. If the launch date is held constant and the same procedure carried out for various arrival dates, there will be a locus of required velocity vectors generated. If the reader can visualize a space determined by a set of three axes calibrated in velocity units, then this locus is a line running through the space with each point on the line corresponding to a specific arrival date. If a velocity increment is applied on the launch date, the spaceship will go to the destination without miss if the tip of the velocity vector lies on the locus. The arrival date is determined by where on the locus the tip of the vector lies. If a velocity increment is applied which does not lie on the locus, the spaceship will have a non-zero miss distance at the destination. If the launch date changes, the line representing the locus of required velocity will move in space. Consequently, at every instant of time there is a moving locus in velocity space which represents the incremental velocity required at that time to get on a free-fall trajectory going to the destination. After launch on an interplanetary trip the same locus still exists at every instant of time. There is some locus of velocity increment required in order to arrive at the destination without miss. Each vector velocity change moves the origin of the axes by that vector amount. When the locus goes through the origin, the spaceship is on a free-fall trajectory to the destination. From the point of view of the spaceship navigator the problem becomes one of defining the required velocity locus and maneuvering to make it go through the origin of the coordinate axes in velocity space. Once the locus goes through the origin, any velocity increment along the locus does not cause a miss at the target but only changes the time of arrival. For this reason the direction of the tangent to the velocity locus is called the non-critical direction because a velocity change in that direction does not contribute to the miss but only changes the time of arrival.

7. The Simplest Case

The simplest possible mode for interplanetary navigation is to assume that the departure planet, the destination planet, and the spaceship are all falling in straight lines through field-free, inertial space. While this is a gross simplification of what is actually happening, it was pointed out earlier that during the times when most of the navigation occurs this is approximately what the situation looks like to the spaceship observer. With this as a justification for such a gross simplification, consider the problem of intercepting a planet moving with constant velocity in field-free space. The velocity space for this problem is shown in the plane of the relative motion just prior to launch in Figure 1. The magnitude and direction of the nominal launch velocity are determined by the nominal time of arrival chosen prior to launch. The spaceship leaves the departure planet by applying the nominal launch velocity as accurately as possible. If this is done perfectly, the spacecraft will contact the arrival planet at the nominal time of arrival without further correction. The line of sight to the departure planet will remain fixed along the original departure direction and the line of sight to the arrival planet will remain fixed along the original line of sight. If the original launch velocity is not applied perfectly, the navigator will assess his situation by observations of the two planets. The best information about the miss at the target will come from the motion of the line of sight to the destination. Just as every good ship captain realizes, constant bearing and closing range are the indications of a collision course. The best information about the time of arrival will come from the direction of the line of sight to the departure planet. If the line of sight to the departure planes is as planned, but the line of sight to the destination drifts, a miss at the target is indicated and a correction is on order. The navigator has the choice of re-establishing the original nominal time of arrival, in which case the velocity correction would have the same direction as the nominal launch velocity, or of minimizing the correction in which case the velocity correction would be perpendicular to the line of sight to the destination. The first choice is known as fixed time of arrival (FTA) navigation and the second choice is known as variable time of arrival (VTA) navigation. When the FTA correction is made, the arrival time existing prior to the correction is changed back to the original nominal arrival time and the line of sight to the departure planet stays fixed. When the VTA correction is made, the arrival time remains unchanged from the value existing prior to the correction. Since this arrival time is different from the original nominal arrival time by virtue of the original error in velocity application, the line of sight to the departure planet will drift to a new direction after the VTA correction. Summarizing the basic principles, the miss at the destination is predicted by observation of the line of sight to the destination and nulled by holding that line of sight constant. The time of arrival is predicted by observation of the line of sight to the departure planet. This line of sight has no tendency to drift unless VTA navigation is used which allows the time of arrival to be different from the pre-planned nominal value in order to minimize the total required velocity correction.

8. Timing the Corrections

The timing of navigational corrections is important because it determines the size and number of corrections and therefore the total fuel required. To develop an understanding of the relationships between the time of a correction and the required velocity, consider again the simplest case in field-free space. The spaceship proceeds toward the destination with range decreasing at a constant rate V. At time t = 0 with remaining time to go, t_g, assume there is a small component of relative velocity perpendicular to the line of sight to the destination of magnitude a V ($a \ll 1$). If uncorrected, the line of sight will drift through an angle A in time t indicating a potential miss at the target. For small A

$$A = a \frac{\frac{t}{t_g}}{1 - \frac{t}{t_g}} \tag{2}$$

A velocity increment, ΔV, applied perpendicular to the line of sight in the direction opposite the drift will null the error at the target.

$$\Delta V = \frac{AV}{\frac{t}{t_g}} = \frac{aV}{1 - \frac{t}{t_g}} \tag{3}$$

Both A and ΔV increase with time as given by (2) and (3). The measurement of A is used to determine ΔV so the error in the applied velocity correction $\epsilon(\Delta V)$ will be due primarily to the error, $\epsilon(A)$, in the measurement of the change of the line of sight.

$$\epsilon(\Delta V) \cong \epsilon(A) \frac{V}{\frac{t}{t_g}} \tag{4}$$

The ratio of (3) to (4) is the ratio of the miss before correction to the miss after a correction at time, t. Note that this ratio increases with time in the same manner as A increases with time. The longer you wait, the larger the required velocity correction becomes. There is no sense, however, in performing a correction until the measured value of (3) is greater than the expected value of (4). Battin[3] has suggested making the expected value of the ratio of the uncorrected miss to the corrected miss the criteria for timing a correction. He has attempted to find the optimum ratio for cislunar trajectories by experiment. Breakwell[4] has suggested making the corrections at regular intervals and gives an analytic argument for performing the correction one third of the remaining distance to the target. On the average this is equivalent to setting the miss reduction ratio because the ratio grows as a function of the time. Breakwell concludes that the total required velocity does not change much with small changes in the ratio. More important than the exact value of the ratio is the observation that the criteria of minimizing the total velocity correction calls for the fractional reduction of the miss in relatively small steps. This means that the model used for determining the correction from the measurements does not need to be extremely accurate. If the velocity correction is 20% in error due to the sighting inaccuracy, it is of little consequence if the correction is 5% in error due to an approximation of the orbital mechanics. For manned missions the timing of the correction will be a command decision made by the captain of the spaceship on the basis of several considerations. Rather than have a predetermined correction schedule, it is more important that the captain understand the physical principles which give rise to the trade-offs.

9. Conclusions

This chapter has intended to show the simple, basic principles involved in interplanetary navigation. Many of the principles are easily visualized in field-free space and this model is justified because the actual forces are small in a non-rotating planet-centered frame during periods when much of the navigation takes place. For actual navigation it is shown that the model of the orbital mechanics does not need to be extremely accurate for determining the correctional velocity from the optical sightings.

The assumption of straight-line motion of the spaceship relative to the reference during planetary homing, for instance, may suffice. Because the limiting factor in the accuracy of navigational corrections is the accuracy of the optical sightings, it will be necessary to know the reference value of the celestial angles to an accuracy better than the accuracy of the sextant. Any phenomena which would cause an error of greater than a few arc-seconds should be accounted for in the determination of the reference angles.

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SECTION VI

GUIDANCE OF LOW-THRUST INTERPLANETARY VEHICLES*

by

Edgar D. Mitchell

The problem studied in this thesis is the guidance of interplanetary vehicles which are thrusting for a large portion of the transfer. The vehicle is represented by a seven component state vector consisting of the position, velocity and mass of the space craft. The analysis is linearized by assuming that the actual state of the vehicle differs only a small amount from a known reference state. The reference trajectory is assumed to be a propellant optimal path connecting the initial and final points.

The goal of the postulated guidance system is to satisfy the position and velocity conditions at the target with minimum propellant expenditure. Both fixed-time-of-arrival and variable-time-of-arrival guidance are discussed. Specification of the guidance criterion in the above manner permits the techniques of optimal control theory to be applied to the problem. Emphasis is placed on finding an analytic solution of the linearized equations. The desired solution is the control program which satisfies boundary conditions and minimizes propellant expenditure.

*Note: This section is the abstract from the Doctor of Science Thesis TE-8 completed in June, 1964, by Edgar D. Mitchell.

The method of solving the guidance problem is shown to be suitable as a technique for computing optimal reference trajectories. The trajectories are computed by iterative application of the guidance solution. Application of the guidance solution to the trajectory problem is shown to exploit an interpretation of the Euler equations which permits simplification of the computation technique.

The guidance solution is tested in a numerical example by using it to compute trajectories from Earth's orbit to the Martian orbit for different low thrust vehicles.

The guidance solution is based on the assumption that vehicle state is known at the time a new control program is to be generated. Prior studies by several investigators detail methods of using celestial measurements to estimate state. A portion of this report is devoted to extending the method of celestial measurements to include measurement of engine performance. The additional measurement is shown to improve the estimate of state.

A discussion is presented of the computational difficulties arising from differences in the criterion for optimality as interpreted from the calculus of variations and from Pontryagin's maximum principle.

SECTION VII

DOCUMENTATION

by

Edward B. Haddad

In early January 1964 it became evident that a specialized library would be necessary to support the functions of the Experimental Astronomy Laboratory. Accordingly, we have set up a library which includes the texts and periodical literature related to our fields of study and interest in the broad area of space navigation. The resulting collection of literature has provided us with ready reference material in both the theoretical and engineering aspects of navigation, guidance systems, and components.

In addition to the material mentioned above, we are establishing a file of manufacturing literature. This information, when compiled, will be a valuable source of information on the current state-of-the-art in components and systems.

We have also subscribed to the available abstracting services (STAR, IAA and DDC) and have established a working contact with the Bedford, Mass., office of DDC for their document search and retrieval proceedures. At present, we are unable to store classified information in our laboratory, but we will have facility clearance on moving to our new quarters in the coming months.

Plans have been made to expand our library services and facilities if it becomes necessary to do so, and we expect to continue to be able to provide all necessary in-house services for efficient documentation.

SECTION VIII

A DIFFERENT CONCEPT OF THE SPACE NAVIGATION PROBLEM*

by

B.E. Blood

I want to describe some analytical work on a different concept of the space navigation problem. It is premature to estimate the full implications and usefulness of this concept; so I will describe the concept and try to indicate the directions which I think the study will take.

It has occurred to me that an examination of the space navigation problem in terms of field theory might lead to useful results. By field theory I do not restrict myself to the classical theory of gravitation; so, to clarify what I mean, I will show a very simple example. In Fig. 1(a) we have a center of attraction C, the source of an inverse square gravitational field. Around this center are two concentric circles drawn at radii specified as follows: A particle on the outer circle with a velocity V_o (normal to the circle) will fall to the center in a time, τ. Also, a particle projected normally out from the inner circle will fall back to C in the same time, τ . I choose to consider these circles as equipotentials of a velocity field. That is, the initial velocity that a particle must have to fall to C in a specified time, τ, can be determined by the gradient of a scalar function of position, the velocity potential. In Fig. 1(b) more circles are drawn to represent other initial velocities. This diagram then, in a way, is a map of the velocities required to complete a particular mission.

*(A talk given at the MIT/NASA briefing of 14 April 1964)

With this in mind, we can take a more complicated model. In Fig. 1(c) the target to be hit is displaced from the center. In this diagram the question is- What form do the velocity potentials take? If we assume that they will be distortions of the circles, we might expect the kind of curves shown in Figs. 1(c) and 1(d). As an aid in trying to find an analytical expression for these curves, I have derived a simple equation. For the two body problem in Fig. 2, it is easy to show that the required initial condition to cause a particle's trajectory to pass through the target P is

$$H_P H_C = Rk(1-\cos\theta) \tag{1}$$

in which

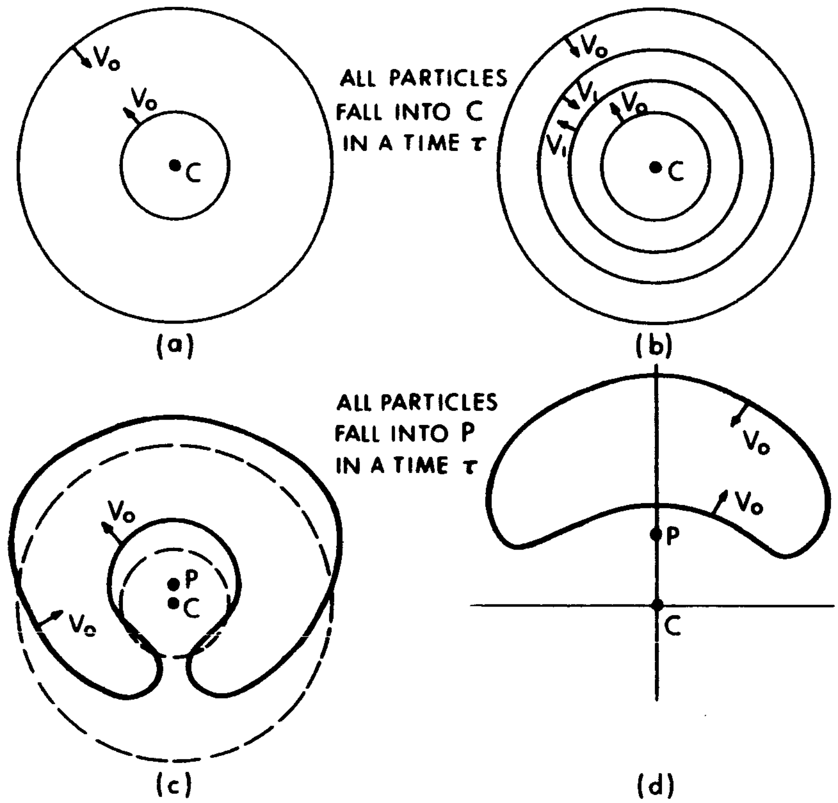
H_P = the massless angular momentum of the particle about the target,

H_C = the massless angular momentum of the particle about the center,

k = the gravitational constant associated with C.

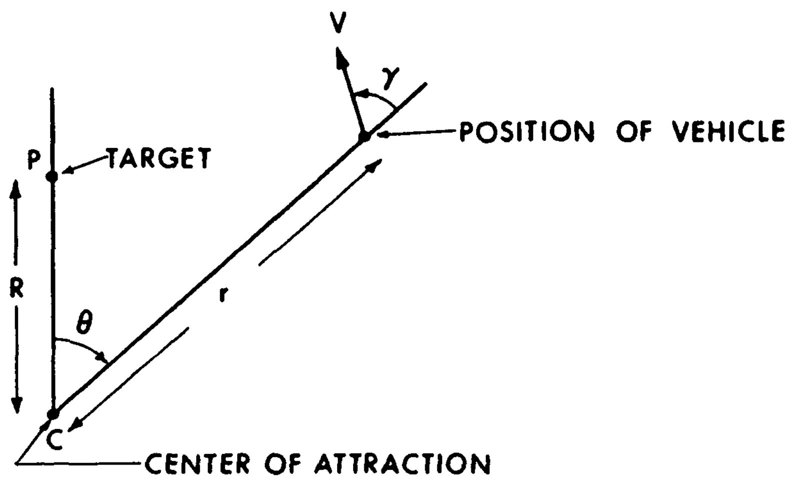
This equation establishes an analytic relation between V and γ in terms of the target's location and the initial position. To complete the solution to the problem, we need another equation which gives an analytic relation between V and γ in terms of a desired time of flight, τ. This is not so simple!

My attack on the time of flight problem was, first, to examine the parameters of the conic trajectories specified by equation (1). (I did this, primarily, because Lambert's equations will give τ in terms of the semimajor axis, the target location, and the initial position.) The initial result of this attack was a discovery that the magnitudes of the semimajor axes of the conics, passing through an initial point r, θ and through the target point, are symmetrical about a value of γ corresponding to the minimum energy trajectory. To illustrate this symmetry I have made use of a parameter δ which is defined in Fig. 3. A plot of the semi-



V_o
V_o
C
ALL PARTICLES
FALL INTO C
IN A TIME τ
V_o
V_1
V_1
V_o
C
(a)
(b)
ALL PARTICLES
FALL INTO P
IN A TIME τ
V_o
P
C
V_o
V_o
V_o
P
C
(c)
(d)

Fig. 1



$H_p H_c = Rk(1-\cos\theta)$

H_p — THE MASSLESS ANGULAR MOMENTUM ABOUT p

H_c — THE MASSLESS ANGULAR MOMENTUM ABOUT c

k — THE GRAVITATIONAL CONSTANT FOR c

Fig. 2 The two body problem in polar coordinates.

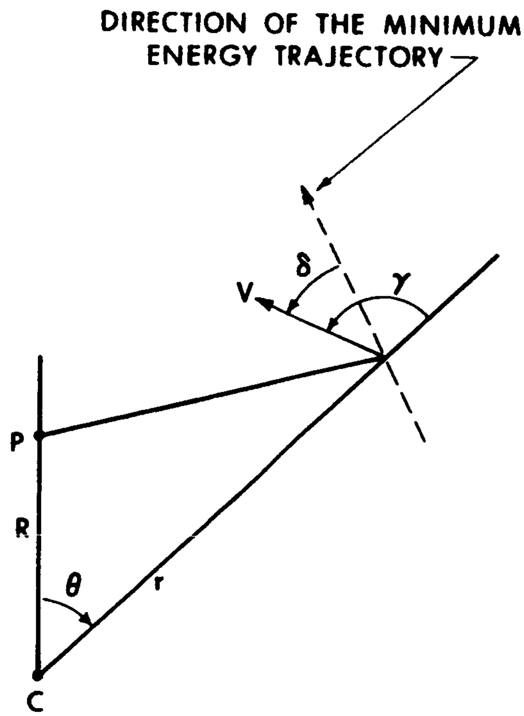


Fig. 3 Definition of δ.

major axis, a, as function of δ is shown in Fig. 4. From the well known relation

$$V^2 = k\left(\frac{2}{r} - \frac{1}{a}\right), \qquad (2)$$

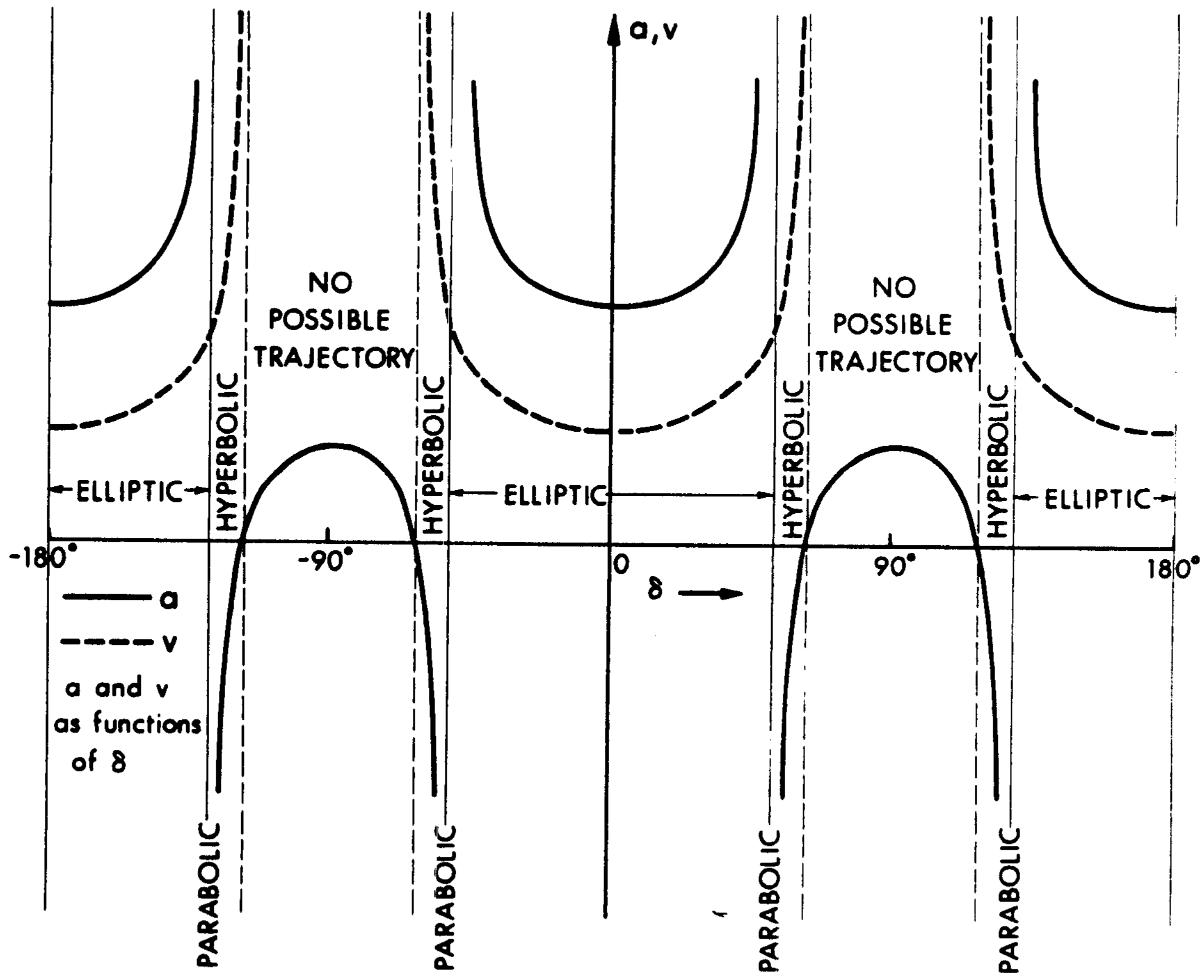
we can also show in Fig. 4 that the velocity, V, is symmetrical about $\delta = 0$. On this diagram we can see the ranges of δ for the various types of conic trajectories. On the left hand side of the diagram the hyperbolic and parabolic trajectories are not permissible paths; they represent outgoing legs of the conics. This situation is further illustrated in Fig. 5.

To return to the problem of finding a relation between V and γ in terms of τ, I chose a numerical example and calculated (using Lambert's equations) τ as a function of δ. This in itself was not a particularly enlightening exercise; however, in a guess that the ratio $\frac{H_P}{H_C}$ was related to τ, I wrote the following expression:

$$\frac{H_P}{H_C}\,\tau = f(\delta) \qquad (r, \theta \text{ fixed}) \qquad (3)$$

Then, using the numbers from a numerical example, I plotted $\log \frac{H_P}{H_C}\,\tau$ as a function of δ. This result is shown in Fig. 6.

Although an analytic proof has not been discovered, it would appear that the curve has a certain symmetry about a line $\delta = \delta_1$. This is an interesting result because δ_1 is the value of δ that splits the range of possible trajectories (as shown in Fig. 5). Although this was an encouraging result, I have not yet extended the work in this direction; so at this point I will turn to a description of another approach to finding the form of the velocity equipotentials.



a,v
NO
POSSIBLE
TRAJECTORY
NO
POSSIBLE
TRAJECTORY
ELLIPTIC
HYPERBOLIC
HYPERBOLIC
ELLIPTIC
HYPERBOLIC
HYPERBOLIC
ELLIPTIC
-180°
-90°
0
90°
180°
δ
a
v
a and v
as functions
of δ
PARABOLIC
PARABOLIC
PARABOLIC
PARABOLIC

Fig. 4

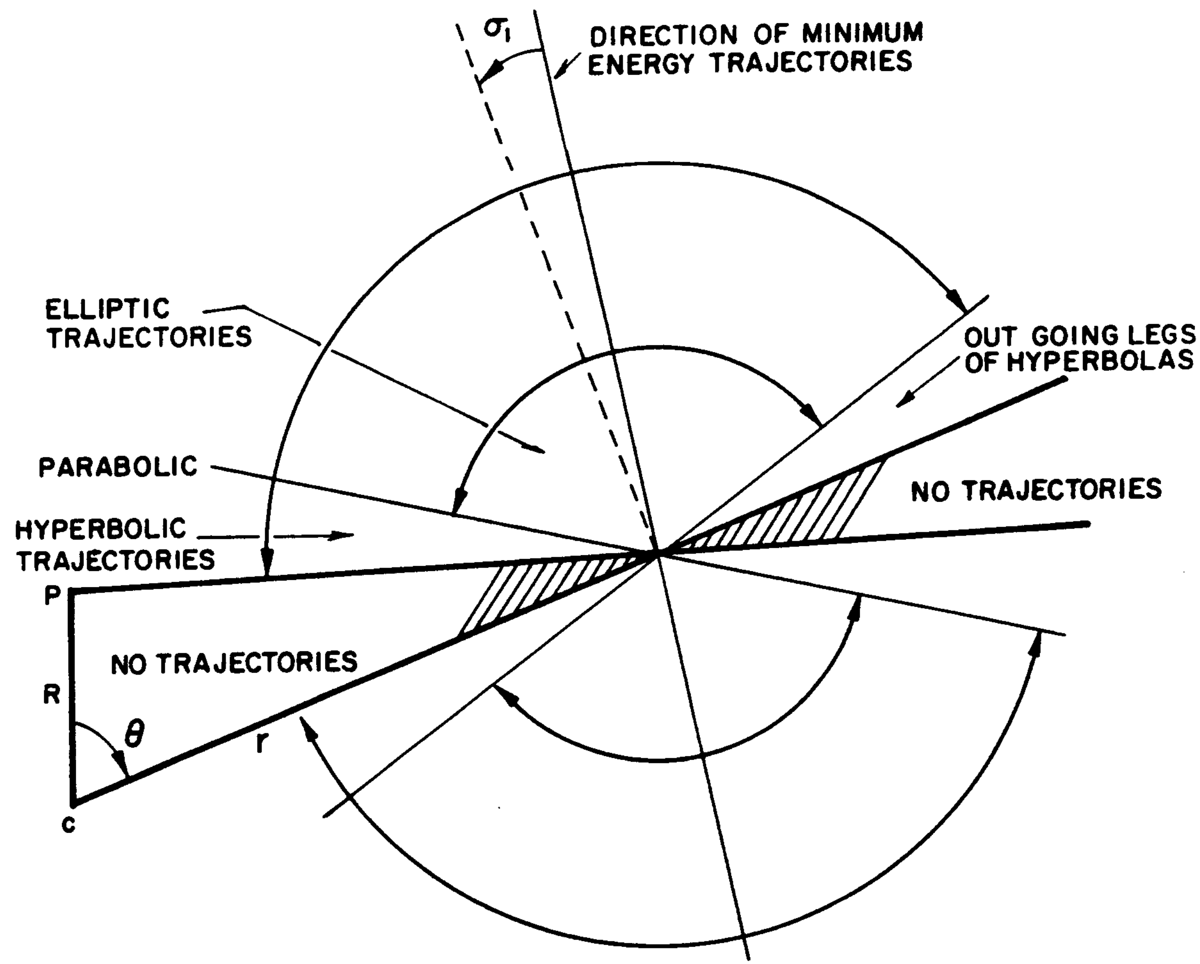


Fig. 5 Conic trajectories from r, θ to P.

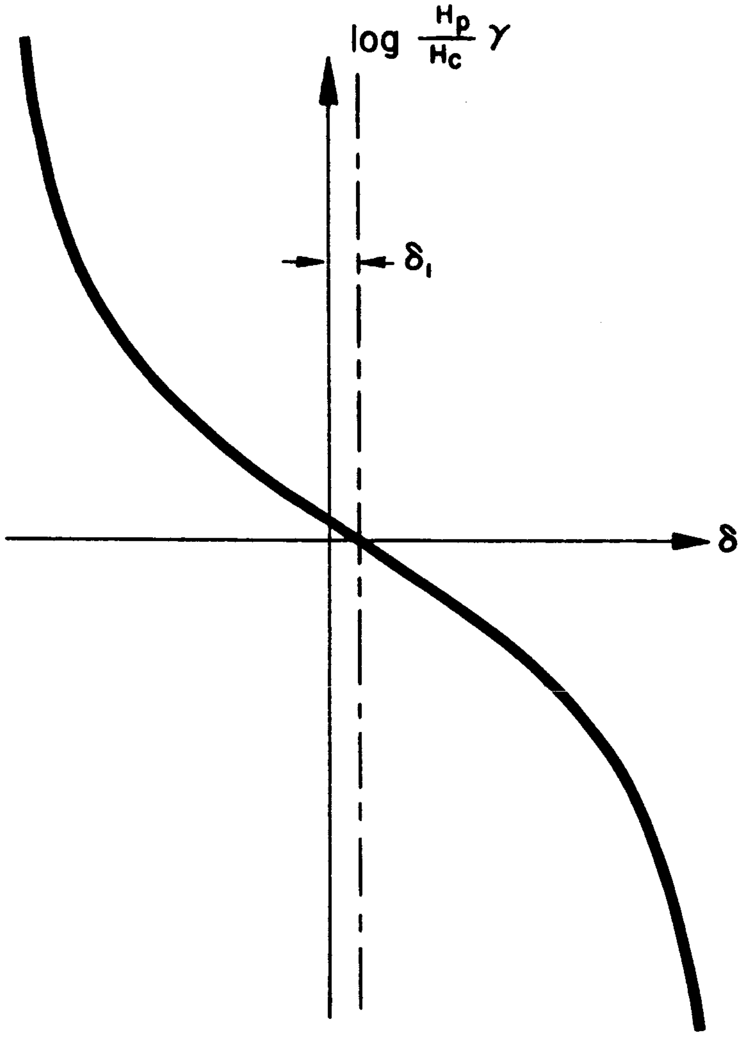


Fig. 6 $\frac{H_p}{H_c}$ as a function of δ.

Under the assumption that the velocity is the gradient of a scalar potential, we write

$$\overline{V} = \nabla\Phi. \tag{4}$$

Now if we take this condition in conjunction with

$$\overline{H}_P \cdot \overline{H}_C = Rk(1 - \cos\theta), \tag{5}$$

we can derive a differential equation.

$$Rk(1 - \cos\theta) = \left(\frac{\partial\Phi}{\partial\theta}\right)^2 \left(1 - \frac{R}{r}\cos\theta\right) + \frac{\partial\Phi}{\partial\theta}\frac{\partial\Phi}{\partial r} R\sin\theta \tag{6}$$

The solution to equation (6) would give us an equation for the curves, Φ = constant; however, after some threshing about, I discovered that a solution is very difficult to find. In an effort to simplify the situation, I derived the ordinary differential equation

$$\left(\frac{d\theta}{dr}\right)^2 - \frac{V^2}{k}\frac{\sin\theta}{(1-\cos\theta)}\frac{d\theta}{dr} + \frac{1}{r^2} - \frac{V^2}{Rk(1-\cos\theta)} + \frac{V^2\cos\theta}{rk(1-\cos\theta)} = 0. \tag{7}$$

In equation (7) V is a constant and the solutions, $\cos\theta$ = some function of r, will be the equipotentials. Again I have been unable to find a general solution. The relation

$$\cos\theta = \frac{r^2 + R^2}{2rR} \tag{8}$$

is a singular solution--valid only at $r = R$. Giving up, for the moment, on finding a general solution for equation (7), I am presently working on a numerical approach in an effort to get some insight for a possible analytical solution.

I want to point out that equations (6) and (7) were derived on the conditions that the particle pass through the target point and the velocity is a gradient of a scalar function of position. This did not include any condition on the time of flight; so one important thing to be done, when I have a numerical solution, is to examine the time of flight for particles on an equipotential.

Finally, I hope this short discussion has conveyed some idea on the potentialities of a field theory approach to the space navigation problem. So far my studies, of simple relations and symmetries, have not yielded any startling results; but, still, I think the concept is very much worth pursuing.

SECTION IX

RESEARCH ON SPECIAL SPACE GUIDANCE PROBLEMS

by

James E. Potter

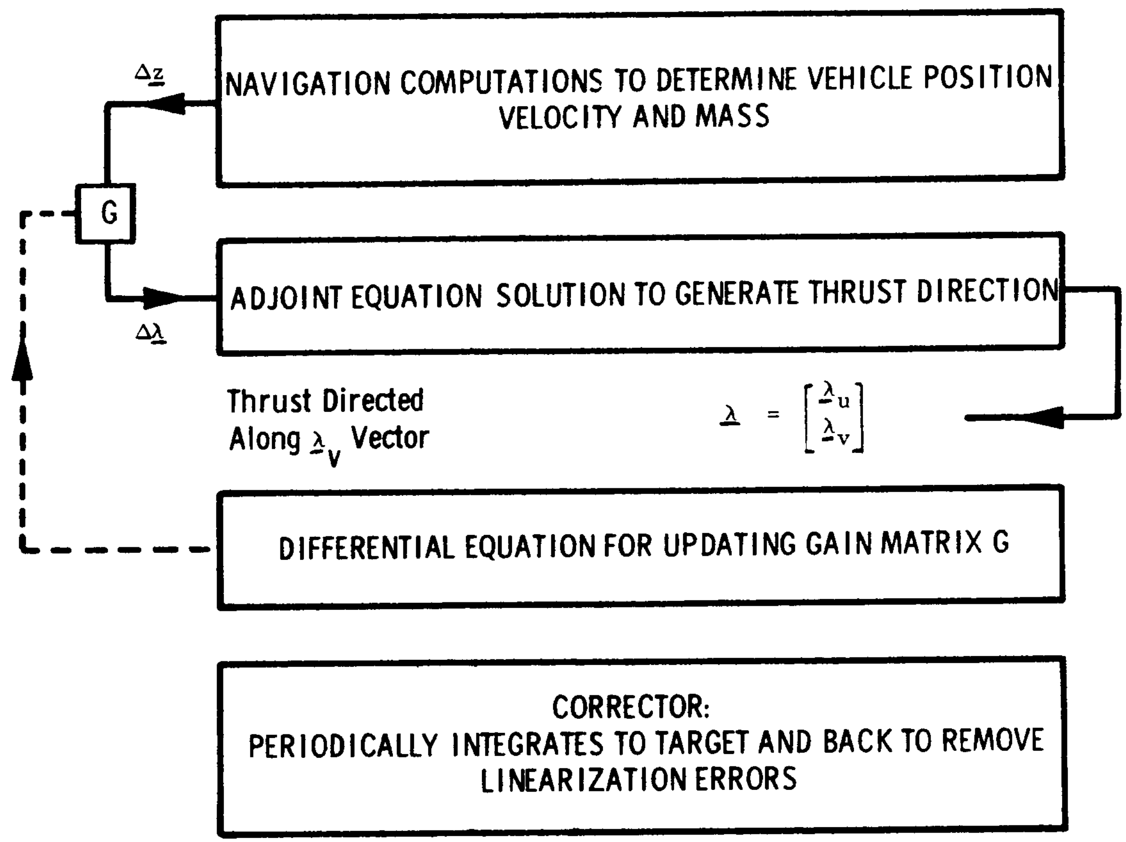
This section presents two areas of space guidance research in which I am working.

1. Continuous Low Thrust Guidance for a Mars Mission

This work is a continuation of Commander Mitchell's thesis work in which a fairly complete digital simulation of the low thrust guidance system (shown in Fig. 1) will be carried out. Low thrust guidance differs from impulsive correction midcourse guidance largely because the thrust direction must be computed continuously. This is done in the proposed system by directing the thrust along the three dimensional velocity Lagrange multiplier vector λ v. This results in minimum fuel expenditure within the limitations of navigational uncertainties. λ v satisfies a differential equation which is solved in real time by the guidance computer. The initial values of the Lagrange multipliers are precomputed for the nominal trajectory to Mars. When navigation measurements indicate that the position and velocity of the spaceship obtained by dead reckoning are in error, the Lagrange multipliers are corrected according to the formula

$$\begin{bmatrix} \Delta \underline{\lambda}\, u \\ \Delta \underline{\lambda}\, v \end{bmatrix} = G \begin{bmatrix} \Delta \underline{u} \\ \Delta \underline{v} \\ \Delta m \end{bmatrix}$$

In this equation u, v and m denote spacecraft position, velocity, and mass, respectively, and G is a gain matrix obtained by solving a matrix Riccati differential equation in real time in the guidance



GUIDANCE SYSTEM FOR CONTINUOUS THRUST VEHICLE

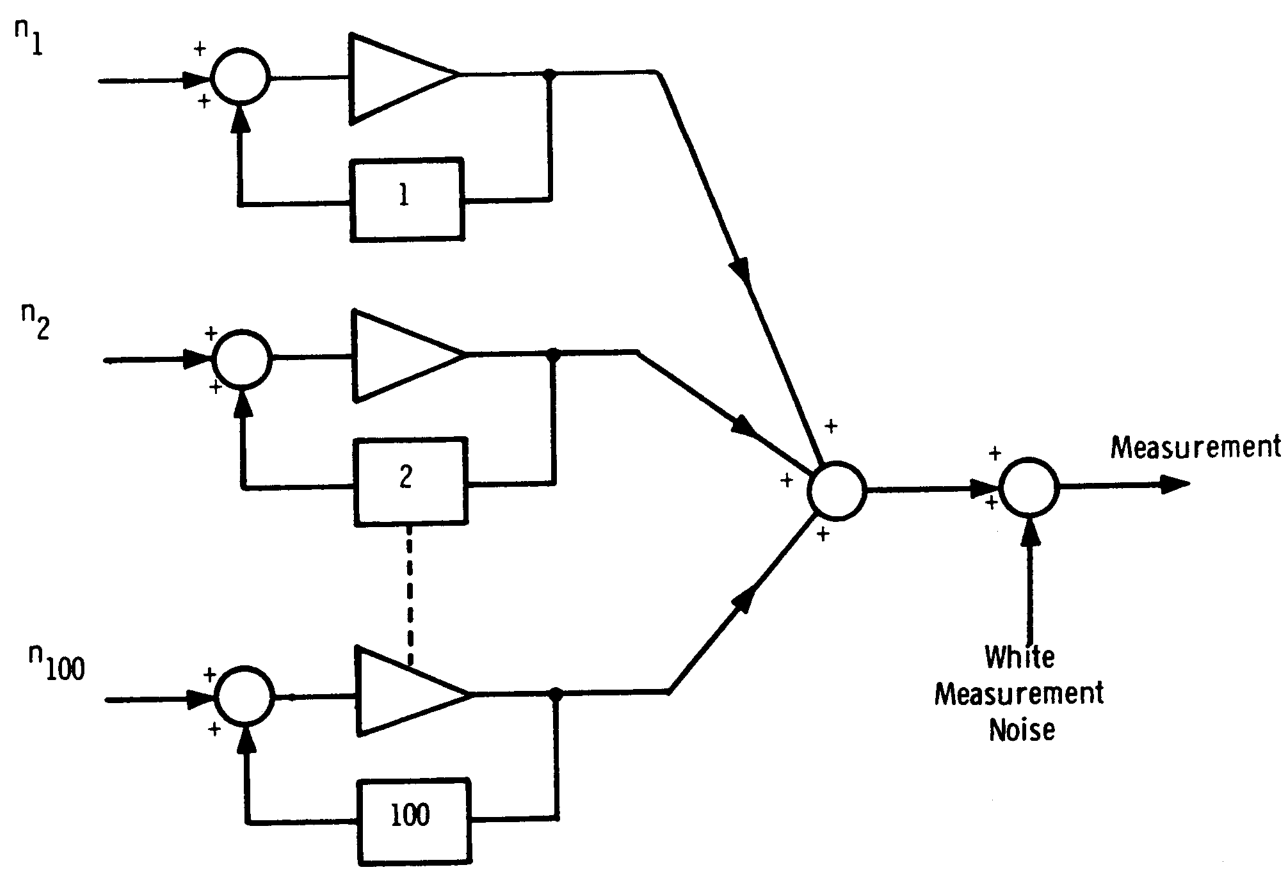
Fig. 1

computer. The same procedure is used to correct the Lagrange multipliers for thrust errors due to engine down time or incorrect steering.

This guidance procedure is computationally an open loop one, since $\Delta\underline{\lambda}$ is not computed directly as a function of target miss. Because of nonlinearities and the high sensitivity to steering errors at the beginning of the mission, it will probably be necessary to close the loop by making a "corrector" calculation several times during the flight. The "corrector" routine integrates forward to the target and then back from the target to the present time. This gives the correct $\underline{\lambda}u$ and $\underline{\lambda}v$ to hit the target, given the present best estimates of $\underline{u}$, $\underline{v}$ and m, and generates a new gain matrix G. The "corrector" calculation is carried out by the guidance computer in its spare time when it is not doing real time computations. This guidance system does not make use of a reference trajectory for in-flight computations.

The guidance system will be simulated digitally using random numbers to simulate navigation sighting errors and inertial platform drift.

Low thrust guidance presents some control problems since after the midcourse coast phase the miss at the target is no longer completely controllable for fixed time of arrival guidance. With variable time of arrival guidance the target miss is still controllable after the coast phase. The linear range is probably quite small, however, since the target miss is not completely controllable after the coast phase even with variable time of arrival guidance in field free space. In view of this, it might be desirable to include a (nonoptimum) coast phase near the sphere of influence to improve controllability. Another problem arises because the gain matrix G becomes infinite as the target is approached.



EXAMPLE OF AN UNSTABLE SYSTEM WHOSE STATE CAN BE ESTIMATED
WITH FINITE STEADY-STATE RMS ERROR

Fig. 2

2. Investigation of the Matrix Riccati Approach to Wiener Filter Theory

As Kalmán pointed out, the matrix Riccati equation approach to statistical filtering allows Wiener filter theory to be generalized to handle the estimation of the output of an unstable system. This is useful in designing a control system which is to be used to stabilize an unstable open loop system. In this case the build-up of uncertainties due to measurement noise is determined by the open loop characteristics of the controlled system and the classical Wiener theory, which only works for stable systems, is not applicable.

In this investigation the conditions under which the rms estimation error approaches a steady state value were studied. Figure 2 gives an example of an unstable system with a one hundred dimensional state vector whose state can be estimated with finite steady state rms error from a single noisy measurement.

This investigation developed a new iterative method for solving matrix quadratic equations. This method may be used instead of spectrum factorization to synthesize Wiener filters, The matrix quadratic method requires the inversion of high order matrices and is therefore of about the same degree of computational difficulty as spectrum factorization. It does have the advantage, however, of working for unstable systems.